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THE PRINCIPLES OF OPTICS

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FIRST EDITION
SIXTH IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK AND LONDON
1932

BC in this figure is equal to one wave length, the disturbance from B will arrive at M_1 exactly one period behind the disturbance from A. Reinforcement will therefore take place. Now, from the geometry of the figure,

$$BC = AB \cdot \sin\alpha.$$

Designating the distance AB by e, for convenience, the condition for reinforcement at M_1 is simply that

$$e \sin\alpha = \lambda.$$

Since all the rulings of the grating are assumed to be separated by the same interval, it is clear that they will all contribute disturbances of the same phase at M_1. An image of the slit will therefore be formed at this point. This would be true also if the distance BC were equal to 2λ, 3λ, or any other integral number of wave lengths. Thus the general condition for reinforcement is

$$e \sin\alpha = m\lambda, \tag{286}$$

where m is any integer. Evidently a number of images, such as M_1, M_2, M_{-1}, M_{-2}, etc., will be formed on both sides of M_0, the limit to the number being set by the fact that $\sin\alpha$ cannot exceed unity.

If the slit is illuminated by white light instead of monochromatic light, it is apparent that light of all wave lengths will be combined at M_0 to form a white image of the source. If now we assume m to be unity, it is clear that, since α varies with λ, two spectra will be formed on opposite sides of M_0, the blue ends lying closer to M_0 than the red ends. These are called the first-order spectra. With $m = 2$, a second pair of spectra are formed, the corresponding portions of which lie at a greater distance from M_0 than those of the first order. The spectra of different orders overlap, of course. For example, the image of a yellow line at 600 mμ in the first order coincides with the line at 300 mμ in the second order, 200 mμ in the third order, etc. This is ordinarily a disadvantage, although it was turned to good use by Rowland in his classical determination of the wave lengths of the lines in the solar spectrum.

The dispersion produced by a grating can be determined by differentiating Eq. (286) with respect to λ, which gives

$$\frac{d\alpha}{d\lambda} = \frac{m}{e} \cdot \frac{1}{\cos\alpha}. \tag{287}$$

It will be seen that the dispersion is approximately proportional to the order of the spectrum and to the fineness of ruling of the grating.

The image of a spectral line formed in the plane of the cross hairs of the telescope or on the photographic plate is, of course, a diffraction pattern whose size depends upon the aperture of the system. Hence, it is clear that the resolving power of a grating depends upon both its size and the dispersion which it produces. A little consideration will show that the resolving power in the spectrum of a given order depends on the total number of lines of the grating that are effective in producing the spectrum. We may therefore write that the resolving power

$$\frac{\lambda}{d\lambda} = Nm, \tag{288}$$

where N is the total number of rulings and m is the order of the spectrum. If we consider the numerical example discussed in Sec. 200, where a prism was required to separate two lines 1 mμ apart at a wave length of 500 mμ, it will be seen that a grating containing only 500 lines will suffice in the first order, one of 250 lines in the second order, etc. Rulings as fine as 20,000 lines per inch are not uncommon, and hence, even in the first order, a grating less than 1.5 mm in size will resolve two lines that are 1 mμ apart.

Another consequence that can be deduced from Eq. (287) is that the spectrum produced by a grating is normal. In any practical application, the angle α is usually small, and consequently the value of $\cos\alpha$ is sensibly constant over a reasonably short angular interval. The value of $d\alpha/d\lambda$ is therefore approximately constant, and the linear separation of two lines in a spectrogram made with a diffraction grating is therefore very nearly proportional to the corresponding difference in wave length. It will be recalled that this is far from being true for the spectrum produced by a prism, which is much extended at the blue end in comparison with the red end.

We have assumed in this discussion that the grating consists of infinitely narrow rulings, but in practice such gratings would transmit an infinitesimal amount of light even if they could be prepared. The grating will function, however, if the rulings have a finite width and even if the intervening spaces are not completely opaque; in fact, the theory just developed holds

equally well if the grating has any sort of periodic structure whatever. The only effect of varying the ratio between the transparent and the opaque portions of the grating is to vary the distribution of light among the various orders of spectra. For example, by making the opaque and transparent portions equally wide, the spectra of even orders disappear.

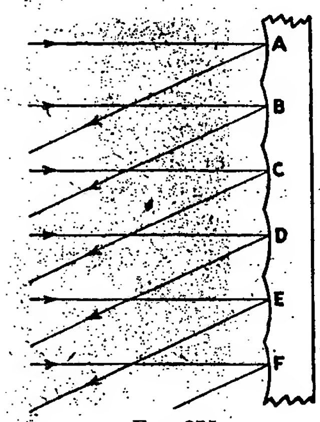

FIG. 275.

To see that the simple theory applies regardless of the type of ruling, let us consider the case of a reflection grating, which consists of a mirror scratched with parallel equidistant lines by means of a diamond point. A cross section of such a grating when enlarged might have the appearance shown in Fig. 275. The light incident upon the grating will be diffracted by each infinitesimal area of a grating element, but in any specified direction and for some point of each element indicated by A, B, C, etc., the disturbance produced in the resultant image will have the same phase as the resultant disturbance from the entire element. Since the simple theory applies to the rays diffracted at A, B, C, etc., it also applies to the rays diffracted by the entire grating. Here again the only effect of the form of the ruling is to vary the distribution of light among the various orders of spectra. Hence, by properly selecting the diamond point with which the grating is ruled, practically all the light can be concentrated in a single order.

The ruling of a grating is done with the aid of a dividing engine, which consists essentially of a carriage that can be moved along a track by means of a very accurate screw. The mechanical difficulties that are involved in the manufacture of a fine grating are of unbelievable magnitude. The most serious fault produced by a dividing engine that is anything less than perfect is a periodic error in the ruling. This may be due to a periodic error in the screw or in some part of the driving machinery. A ruling containing a periodic error produces not only the spectra characteristic of the nominal grating interval but also spectra corresponding to the interval of the periodic error. Such false spectra are termed *ghosts*. They can be detected by placing the grating in a spectroscope as usual, substituting a hole for the slit, and placing a prism against the grating with its refracting edge perpendicular to the ruling of the grating. The prism draws out each spectrum into a curve similar to the dispersion curve of the glass, and a little consideration will show that any images of the pinhole that do not lie upon this curve are due to ghosts produced by the grating.

A few gratings with 50,000 lines per inch have been ruled and at least one with 100,000 lines per inch. The ghosts in such fine gratings are so pronounced, however, that better results are obtained with coarser rulings. Most of the work on the fine structure of spectral lines is being done with gratings having 15,000 lines per inch. Such a grating 8 in. long, used in the fifth order, has a theoretical resolving power of 600,000. In other words, at 4000 Å., two lines 0.007 Å. apart should just be resolved.

The cost of even a moderately good grating is very high, but for many purposes replicas made by Thorp's process as modified by Wallace[1] are quite satisfactory. This process consists in pouring a layer of collodion upon an ordinary reflection grating, stripping it from the latter when it is dry, and mounting it between glass plates. The possibility of injuring a replica during the process of manufacture is considerable, and the price varies according to the quality.

Nowadays, reflection gratings are more widely used than transmission gratings.[2] If a grating is ruled on a concave mirror, it can be made to serve as its own collimator and camera objective. This was done by Rowland in 1882, and it was by means of such a grating that he made his classical determination of the wave lengths of the Fraunhofer lines in the solar spectrum. The essential features of Rowland's mounting[3] are shown in Fig. 276. The grating G forms an arc of a circle AGB (really a cap of a sphere of which AGB is the trace). The slit S and the photographic plate P lie on a circle SGP whose diameter is equal to the radius of curvature of the grating. It can be shown that the spectra also lie upon this circle. Sharp

[1] *Astrophys. Jour.*, **22**, 123 (1905).

[2] Replicas can be coated with platinum by cathodic sputtering for use by reflection.

[3] For the details of construction and adjustment, see a paper by Ames in *Phil. Mag.*, **27**, 369 (1889).